and Mozer. Therefore different results may be produced by annealing. In any case, we are only in a position to note the agreement between our results for $\Theta_s$ and that of Maradudin *et al.*[5] and conclude that $\Theta_s$ does not seem to depend on the details of sample preparation. We are in no position to make any detailed assumptions concerning the condition of the $Co^{57}$ in the Be lattice.

Our value of $\Theta_f$ for Cu is in reasonably good agreement with that (360°K) of Dyson.[15] It is to be noted that Dyson prepared his source by melting the copper, whereas ours was obtained by diffusion.

[15] N. A. Dyson, Phys. Letters **1**, 275 (1962).

PHYSICAL REVIEW VOLUME 133, NUMBER 6A 16 MARCH 1964

# Dipole-Narrowed Inhomogeneously Broadened Lines in Ferromagnetic Thin Films*

P. E. WIGEN
*Lockheed Research Laboratories, Palo Alto, California*
(Received 11 October 1963)

An inhomogeneity in the magnetization of thin ferromagnetic films is shown to provide a mechanism for two-magnon scattering which contributes to the spin-wave resonance linewidth of thin ferromagnetic metal films. A normal (perpendicular) variation in the internal field has previously been assumed to explain the spinwave-resonance properties of thin films. In this work a planar variation in the magnetization is assumed which produces an inhomogeneously broadened linewidth. Since the inhomogeneity is much smaller than the average magnetization of the material, dipole forces will decrease the inhomogeneous broadening of the resonance line. Experimental results are in good agreement with the theory when eddy current losses are included and the inhomogeneity in the plane of the film is of the same order of magnitude as the normal inhomogeneity. As the film normal is rotated with respect to the magnetic field direction, the critical angle is approached at which the resonance conditions are the same throughout the film and the spins precess at a common frequency. At this angle the planar inhomogeneity will not be effective in broadening the excited mode. This effect is indicated in the data by a minimum in the frequency linewidth at this angle.

## I. INTRODUCTION

THE inhomogeneous magnetization model[1,2] has been successful in accounting for many properties of thin-film spin-wave resonances.[3,4] It is assumed in this model that the inhomogeneity produces an internal field that is a maximum at the central plane of the film and decreases in a near parabolic manner to each surface (normal inhomogeneity). From an analysis of the data the total change in the internal field between the central plane and the film surface is $4\pi\Delta M \simeq 10^3$ G (Ref. 2). It is unlikely that this normal inhomogeneity will have the same properties everywhere in the plane of the film, i.e., there may be an inhomogeneity in the plane of the film (planar inhomogeneity). While the normal inhomogeneity will determine the position of the modes, the planar inhomogeneity will contribute to the line-width of the resonance modes.

In this paper, the planar inhomogeneity is assumed to be a dipolar disorder which produces a dipole-narrowed inhomogeneously broadened line. The linewidths predicted by this two-magnon scattering process will be compared with the linewidths obtained from a $2.8\times10^3$ Å 80–20 Permalloy film at 9.6 and 14.6 kMc/sec.

The effect of inhomogeneities in the exchange field through a pseudodipolar interaction has been considered in detail by Clogston *et al.*[5] for ferromagnetic insulators. These field inhomogeneities arise from the disorder in the magnetic lattice through the random distribution of metal ions in the octahedral sites of the inverted spinel-structure ferrites and have a spatial periodicity comparable to the distance between spins. For this case the exchange field is the effective agent in narrowing the resonance line.

Geschwind and Clogston[6] have extended this mechanism to a disorder in the ordinary dipole interaction of polycrystalline ferromagnetic insulators, and find that under certain circumstances the dipole forces can reduce the inhomogeneous broadening of the magnetic resonance line. Here, as in thin ferromagnetic films, the inhomogeneity in the internal field is of such a spatial variation that the short-range exchange force coupling cannot tie together the motion of the spins. However, the long-range forces of the ordinary dipole interaction will become an effective agent in narrowing the res-

* Supported by the Lockheed Independent Research Program.
[1] P. E. Wigen, C. F. Kooi, M. R. Shanabarger, and T. D. Rossing, Phys. Rev. Letters **9**, 206 (1962).
[2] A. M. Portis, Appl. Phys. Letters **2**, 69 (1963).
[3] P. E. Wigen, C. F. Kooi, M. R. Shanabarger, U. K. Cummings, and M. E. Baldwin, J. Appl. Phys. **34**, 1137 (1963).
[4] C. F. Kooi, Phys. Rev. **131**, 1070 (1963).
[5] A. M. Clogston, H. Suhl, L. R. Walker, and P. W. Anderson, Phys. Chem. Solids **1**, 129 (1956).
[6] S. Geschwind and A. M. Clogston, Phys. Rev. **108**, 49 (1957).

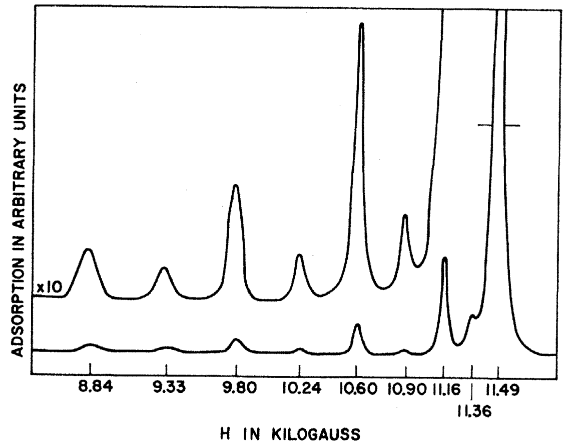


FIG. 1. Spin-wave spectra of a Permalloy film $2.8\times10^3$ Å thick (PC26-3 of Table II). $4\pi M=8.06$ G and $g=2.03$. The frequency was 9.6 kMc/sec. The ticks at each side of the first mode indicate the half-peak power intensity points.

onance line. The condition necessary for such a process is that the inhomogeneity in the dc field be less than the total dipole field.

In thin ferromagnetic films, linewidth contributions due to eddy current effects depend on the inverse square of the spin-wave wave number.[7,8] Theoretical values at 10 kMc/sec predict a linewidth of approximately 50 G for the first mode in a Permalloy film 3000 Å thick. This accounts for about one-half of the linewidth of the first mode at 10 kMc while its contribution to higher order modes decreases as the inverse square of that mode number (see Table I).

As indicated by the data in Figs. 1 and 2 and in other spectra,[9] the experimental linewidths of the first two or three modes decrease while higher order modes increase in linewidth. The decrease in the linewidth of the lower order modes can be attributed to the contribution of eddy-current effects while the increasing linewidths observed in the higher order modes can be attributed to the effect of the planar inhomogeneity of the internal fields.

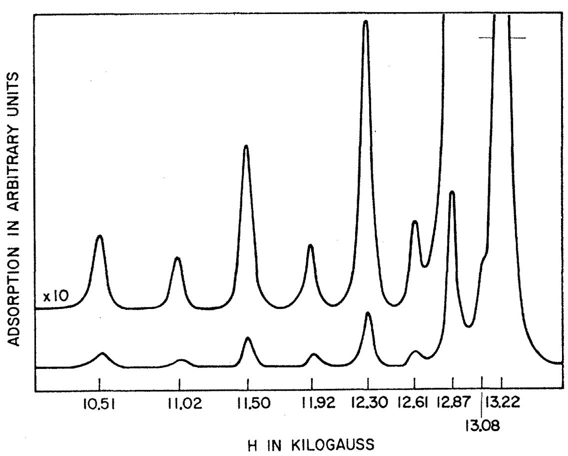


FIG. 2. Spin-wave spectra obtained at 14.6 kMc/sec for the same film used to obtain Fig. 1.

## II. THEORY

The theory of the dipole narrowing effect of inhomogeneities in ferromagnetic metal films can be developed in a manner similar to the theory of ferromagnetic linewidth due to inhomogeneities in the pseudodipolar[5] and ordinary dipolar[6] fields of ellipsoidal ferrites. As discussed in these references, the dispersion relation of a ferromagnetic ellipsoid placed in a uniform magnetic field is given by

$$\omega_\kappa=\gamma[(H-N_zM+H_ea^2\kappa^2)\times(H-N_zM+H_ea^2\kappa^2+4\pi M\sin^2\theta_\kappa)]^{1/2}, \quad (1)$$

where $\omega_\kappa$ is the frequency of a spin wave of wave number $\kappa$, $a$ is the distance between spins, $\gamma$ equals $-g\beta/\hbar$, $H$ is the external magnetic field, $N_z$ is the longitudinal demagnetization factor, $M$ is the magnetization of the sample, $H_e$ is the exchange field, and $\cos\theta_\kappa=\kappa_z/\kappa$. The frequency of the uniform precession, or infinite wavelength mode, $\kappa=0$, is given by the Kittel relation

$$\omega_0=\gamma[H-(N_z-N_t)M], \quad (2)$$

where $N_t$ is the transverse demagnetization factor.

The spectrum given by Eqs. (1) and (2) is shown in Fig. 3. The frequencies $\omega_2$ and $\omega_1$ represent the limiting values of the spin-wave manifold as $\kappa$ approaches zero and are given by

$$\omega_2=\gamma[(H-N_zM)(H-N_zM+4\pi M)]^{1/2}, \quad (3)$$

and

$$\omega_1=\gamma(H-N_zM). \quad (4)$$

In thin ferromagnetic films, the uniform precession mode is not excited. Rather, the normal inhomogeneity defines a new set of $\theta_\kappa=0$ nonplane-wave spin-wave states $\psi_n(z)$, with the $z$ dependence of the oscillating transverse magnetic moment having approximately the same spacial dependence as the $n$th harmonic oscillator wave function.[2] The dispersion field $H_{n,\kappa}$ of the modes is defined as $H_{n,\kappa}=H_0-H_n$, where $H_0$ is $\gamma\omega_0$ of Eq. (2) with $N_t=0$ and $H_n$ is the field at which the $n$th mode is excited. For the pure harmonic oscillator modes,[2] this dispersion field will be linear in the mode number $n$ rather than having the quadratic form of pure plane-wave modes.[7] However, the spatial variation of the transverse magnetization in these modes is very similar to that of a plane wave and in this work a mean

[7] C. Kittel, Phys. Rev. **110**, 836 (1958).
[8] P. Pincus, Phys. Rev. **118**, 658 (1960).
[9] Other examples of spin-wave spectra can be found in works by: C. F. Kooi, W. R. Holmquist, P. E. Wigen, and J. T. Doherty, J. Phys. Soc. Japan **17**, Suppl. B-I, 599 (1962); M. H. Seavy, Jr. and P. E. Tannenwald, J. Appl. Phys. **30**, 227S (1959); P. E. Tannenwald and R. Weber, Phys. Rev. **121**, 715 (1961); Z. Frait and M. Ondris, Czech. J. Phys. **B11**, 463 and 883 (1961); H. Nose, J. Phys. Soc. Japan **16**, 2475 (1961).

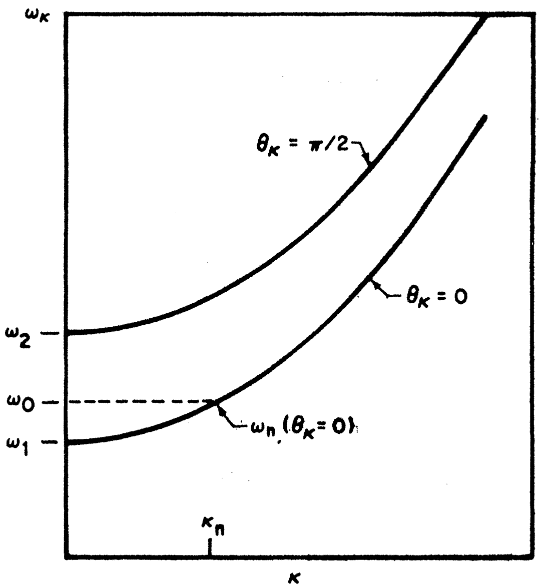


FIG. 3. Dispersion relation for a finite body of ellipsoidal shape with Zeeman, exchange, and dipole-dipole energies included.

propagation constant $\kappa_n$ for the $n$th mode is defined such that $H_{n,\kappa}=H_e a^2\kappa_n^2$ for $\theta_\kappa=0$ in Eq. (1), but $\kappa_n$ will not be related to the wavelength of that mode. The excitation of the various spin-wave modes is similar to changing the shape of the ellipsoidal ferrite and the analog of Eq. (2) for the various modes in a thin film becomes

$$\omega_n(\theta_\kappa=0)=\gamma[H-4\pi M+H_{n,\kappa}]. \quad (5)$$

Using these assumptions, the results of the plane-wave analysis can be applied to thin films.

The important features of Eqs. (1) and (2) including the degeneracy of $\omega_0$ (as well as $\omega_n$) with the manifold of spin-wave modes, have been discussed in Ref. 5. There a perturbation that produced a coupling between the spin wave $\kappa=0$ and a manifold of degenerate spin waves of higher $\kappa$ values was imposed upon the spin system. The specialized assumptions made about the perturbation in the internal field rendered the coupling independent of the magnitude of $\kappa$. The linewidth resulting from this perturbation is given in Eq. (29) of Ref. 5 as

$$\Delta H=(3/20\pi)[H_p^2(4\pi M)^{1/2}/H_e^{3/2}]I(N_t, H/4\pi M), \quad (6)$$

where $I$ is a "shape factor" whose value depends upon the magnetization of the sample and the frequency used in the experiment and varies between 1 and 2 for a sphere to zero for a flat disk. $H_p$ is the root-mean-square value of the field inhomogeneity expressed in oersteds. Since $N_t$ is zero in thin films, the pseudodipolar inhomogeneity will not contribute to the linewidth.

The result of confining the perturbation to the long-range fluctuations in thin films is similar to the analysis in Ref. 6. Using methods similar to those in Refs. 5 and 6, the linewidth for those modes with $H_{n,\kappa}\leqslant 4\pi M$ is given by

$$\Delta H_J=\Delta H_u J_\kappa, \quad (7)$$

where $\Delta H_u$ is the contribution of the scattering mechanism to the uniform precession mode ($J_\kappa=1$) and is related to the planar inhomogeneity by the relation

$$\Delta H_u=(18\pi/10)(H_p^2/4\pi M). \quad (8)$$

$J_\kappa$ depends on the magnetization, frequency, and the mode number $n$ by the relation

$$6J_\kappa=(\omega_m/\omega_1)[\omega_1\omega_n/(\omega_2^2-\omega_n^2)]^{1/2} \times\{1+[5(\omega_n^2+\omega_1^2)/2\omega_n\omega_1]\}, \quad (9)$$

where $\omega_m=4\pi\gamma M$, and $\omega_n$ is given by Eq. (5). The linewidth expected from the root mean-square value of the planar field inhomogeneity $H_p$ is thus narrowed by ordinary dipole forces alone to give a linewidth of order $H_p^2/4\pi M$.

Equation (7) has the same form as Eq. (7) of Ref. 6 where $\omega_n$ has replaced $\omega_0$. Reference 6 considers the scattering of the uniform precession mode to long-wavelength spin waves with $H_e a^2\kappa^2<4\pi M$. This work considers the scattering of the $\theta_\kappa=0$ standing modes with $H_{n,\kappa}<4\pi M$ to $\theta_\kappa\neq 0$ spin-wave modes by the same mechanism.

In comparing Eq. (7) with Eq. (6) the following observations will be of interest.

(1) Equation (6) was developed to deal with rapid spin-to-spin variation of the pseudodipolar forces in a ferromagnet and shows a very great narrowing brought about by the exchange field. Equation (7) considers a more slowly varying perturbation in Eqs. (26)–(28) of Ref. 5 which results in a resonance line that is narrowed only by ordinary dipole forces, and therefore to a much smaller degree.

(2) The shape factor $I$ in Eq. (6) arises from a change in the number of $\omega_\kappa$ states degenerate with $\omega_0$ as a change in the sample shape changes the transverse demagnetization factor $N_t$ in Eq. (2). In thin films $N_t$ is zero. Equation (7) considers the dependence of the linewidth on the number of $\omega_\kappa$ states that are degenerate with $\omega_n(\theta_\kappa=0)$ of the $n$th excited spin-wave mode [see Eq. (24) of Ref. 5].

(3) Equation (6) describes the scattering of the uniform precession mode to spin waves having a dominant wavelength on the order of the lattice constant, while Eq. (7) describes the scattering of the spin-wave mode $\omega_n(\theta_\kappa=0)$ excited in the resonance to spin waves of longer wavelengths [Eq. (24) of Ref. 5].

### III. EXPERIMENTAL RESULTS

The dependence of $J_\kappa$ upon $H_{n,\kappa}$ in Eq. (7) is plotted in Fig. 4 for the conditions $4\pi M=10\,000$ G (bulk values of 80–20 Permalloy) and the frequencies 9.6 and 14.6 kMc/sec. Equation (7) was derived with the assumption that the dispersion field $H_{n,\kappa}$ of the excited

TABLE I. The calculated linewidth $\Delta H_{\text{calc}}$, composed of contributions due to inhomogeneity $\Delta H_J$, and the eddy current contribution $\Delta H_{ec}$, is compared with the experimental linewidth at 9.6 and 14.6 kMc/sec. The inhomogeneity contribution $\Delta H_J$ is normalized to $J_\kappa$ for the fifth mode at 9.6 kMc/sec.

| 9.6 kMc/sec | | | | | | | 14.6 kMc/sec | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_n$ kG | $H_{n,\kappa}$ kG | $J_\kappa$ | $\Delta H_J$ G | $\Delta H_{ec}$ G | $\Delta H_{\text{calc}}$ G | $\Delta H_{\text{exp}}\pm 5$ G | $H_n$ kG | $H_{n,\kappa}$ kG | $J_\kappa$ | $\Delta H_J$ G | $\Delta H_{ec}$ G | $\Delta H_{\text{calc}}$ G | $\Delta H_{\text{exp}}\pm 5$ G |
| 11.49 | 0.06 | 1.02 | 48 | 42 | 90 | 90 | 13.22 | 0.06 | 1.02 | 48 | 62 | 110 | 105 |
| 11.36 | 0.19 | 1.09 | 51 | 11 | 62 | ... | 13.08 | 0.20 | 1.07 | 50 | 16 | 66 | ... |
| 11.16 | 0.39 | 1.17 | 55 | 5 | 60 | 60 | 12.87 | 0.41 | 1.13 | 53 | 7 | 60 | 60 |
| 10.90 | 0.65 | 1.28 | 60 | 3 | 63 | 60 | 12.61 | 0.67 | 1.20 | 56 | 4 | 60 | 60 |
| 10.60 | 0.95 | **1.49** | **70** | 2 | 72 | **70** | 12.30 | 0.98 | 1.36 | 64 | 3 | 67 | 70 |
| 10.24 | 1.31 | 1.70 | 80 | 1 | 81 | 80 | 11.92 | 1.36 | 1.53 | 72 | 2 | 74 | 75 |
| 9.80 | 1.75 | 2.05 | 96 | 1 | 97 | 105 | 11.50 | 1.78 | 1.80 | 85 | 1 | 86 | 85 |
| 9.33 | 2.22 | 2.70 | 127 | ... | 127 | 120 | 11.02 | 2.26 | 2.22 | 104 | 1 | 105 | 100 |
| 8.84 | 2.71 | 3.86 | 181 | ... | 181 | 150 | 10.51 | 2.77 | 2.92 | 137 | ... | 137 | 130 |

spin-wave mode, is less than the dipole field $4\pi M$. Since $H_{n,\kappa}$ in the excited mode is limited by $\omega/\gamma$ of the excitation source, Eq. (7) will be valid through these frequency ranges.

The linewidths calculated from the conditions considered in Fig. 4 are compared with the experimental results in Table I. $H_n$ in the first column is the value of the external field at which the $n$th mode is excited. $H_{n,\kappa}$ in the second column is the dispersion field given by $H_0-H_n$. In column three $J_\kappa$ is determined from $H_{n,\kappa}$ and Fig. 4.

Column four gives the linewidth contribution of the scattering mechanism $\Delta H_J$. In Eq. (8), $H_p$, the root mean-square value of the planar field inhomogeneity, can be determined by normalizing the linewidth to $J_\kappa$ at one of the modes. The fifth mode was chosen for this normalization because the eddy current contribution is small while the mode intensity is strong enough to allow an accurate linewidth determination.

The fifth column of Table I indicates the eddy-current contributions to the linewidths where the remaining contribution of the linewidth to the first mode

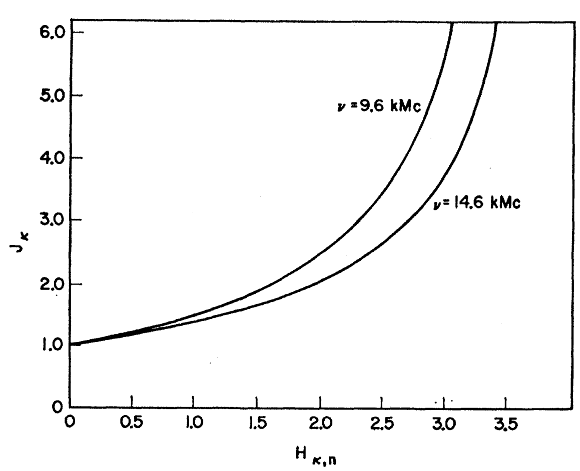


FIG. 4. The dependence of the spin-wave factor $J_\kappa$ on the field $H_{n,\kappa}=H_0-H_n$ for $4\pi M=10\,000$ G (Permalloy).

is assumed to be due to eddy-current losses. The contributions to higher order modes are approximated by assuming that the first mode is a pure cosine wave of wavelength $2L$ and by using the method of Kittel.[7] Since the eddy-current linewidth effect arises from the reciprocal of a $\nabla^2 H$ term in the solution of the equation of motion including the Maxwell equations, the contribution to the higher order modes is better approximated by the inverse square of the mode number than by the square of the mean spin-wave wave number which happens to be equivalent for the conditions considered in Ref. 7. Consequently the contribution of eddy-current losses to the linewidth of the higher order modes is proportional to the inverse square of that mode number.

Column six gives the total calculated linewidth from both the eddy-current and inhomogeneity contributions and column seven gives the linewidths obtained from the experiment.

The calculation is repeated for 14.6 kMc/sec where the same value of $\Delta H_u$ is used and the eddy-current contributions are multiplied by the appropriate frequency dependence. The total calculated linewidth is then indicated in column thirteen with the experimental values given in column fourteen.

From this data $\Delta H_u$ is 47 G. Using the value $4\pi\Delta M = 10^3$ G as the maximum deviation of the normal field, the mean-square value of the normal inhomo-

TABLE II. Summary of the analysis of a number of Permalloy films whose thicknesses vary from 2500 to 3000 Å.

| Film | $T_s$ °C | $L$ Å | $4\pi M$ G | $\Delta H_u$ G | $\rho$ $\mu\Omega$ cm | Ratio |
|---|---|---|---|---|---|---|
| PC22-2 | 75 | 2900 | 8690 | 33 | 15.5 | 2.13 |
| PK15-2 | 100 | 3000 | 8350 | 51 | 24.9 | 2.05 |
| PC20-3 | 150 | 2500 | 8640 | 42 | 18.9 | 2.22 |
| PC19-3 | 200 | 2800 | 9020 | 53 | 27.1 | 1.96 |
| PC26-3 | 200 | 2800 | 8060 | 47 | 21.7 | 2.16 |
| PK17-6 | 200 | 2500 | 8710 | 50 | 20.4 | 2.46 |
| PK22-6 | 225 | 2200 | 10400 | 46 | 22.3 | 2.06 |
| PC18-3 | 250 | 2900 | 9340 | 45 | 19.5 | 2.31 |
| PK18-2 | 250 | 2500 | 9420 | 41 | 16.6 | 2.47 |

TABLE III. Summary of the analysis of a Permalloy film 5000 Å thick. The field values $H_n$ include only the even modes where the first mode is identified by $n=0$ (Ref. 2).

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $H_n$ | 14.18 | 13.96 | 13.77 | 13.53 | 13.24 | 12.88 | 12.46 | 11.98 | 11.40 |
| $\Delta H_J$ | 48 | 51 | 55 | 61 | 69 | 84 | 93 | 130 | 220 |
| $\Delta H_{\text{e.c.}}$ | 82 | 9 | 3 | 2 | 1 | 1 | ... | ... | ... |
| $\Delta H_{\text{Th}}$ | 130 | 60 | 58 | 63 | 70 | 85 | 93 | 130 | 220 |
| $\Delta H_{\text{exp}}$ | 130 | 80 | 55 | 50 | 60 | 60 | 65 | 85 | 105 |
| | | $\Delta H_u=47$ G | | | | $\rho=53.0\ \mu\Omega$ cm | | | |

geneity will be $H_p{}^2=10^5$ G and $\Delta H_u$ will have the value 56 G. The field inhomogeneity in the plane of the film thus appears to be of the same order of magnitude as the normal field inhomogeneity.

Table II summarizes the analysis of several Permalloy films having thicknesses that vary from 2500 to 3000 Å. The spectra of the films chosen for these analyses contained at least seven well-resolved modes, a condition that assures a film with reasonably well understood internal fields.[1–4]

The fifth column indicates the inhomogeneity contribution $\Delta H_u$ of Eq. (8). This contribution varies from 33 to 53 G, or about a factor of two. The sixth column indicates the resistivity of the films determed from the eddy-current contribution[7] and varies from 15.5 $\mu\Omega$ cm to 27 $\mu\Omega$ cm, again about a factor of two. While these numbers appear to be independent of $4\pi M$ or substrate temperature at deposition, the ratio of the two numbers is 2.2±10%. This suggests that the scattering mechanism which contributes to the magnetic resonance linewidth also contributes to the conduction electron scattering mechanism.

While this scattering mechanism gives a quantitative description of the relaxation mechanism for films less than 3500 Å, thicker films give only a qualitative agreement. Typical results are shown in Table III for a film 5000 Å thick. The linewidths of the higher order modes do increase but not as rapidly as predicted by the

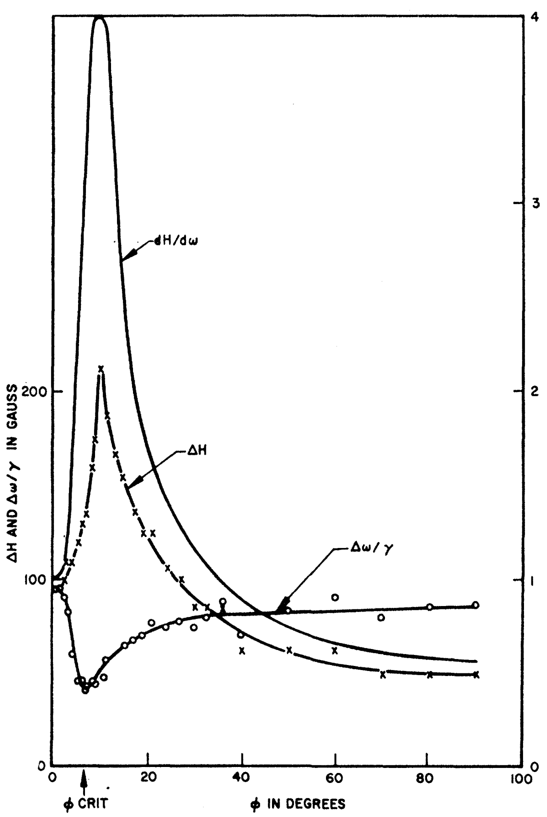


FIG. 5. The angular dependence of the experimental linewidth $\Delta H$, the calculated form of $dH/d\omega$, and the resulting frequency linewidth parameter $\Delta\omega/\gamma$ as the film is rotated from a position with the magnetic field applied perpendicular to the film ($\varphi=90°$). The data are obtained at 9.6 kMc/sec from film PC26-3.

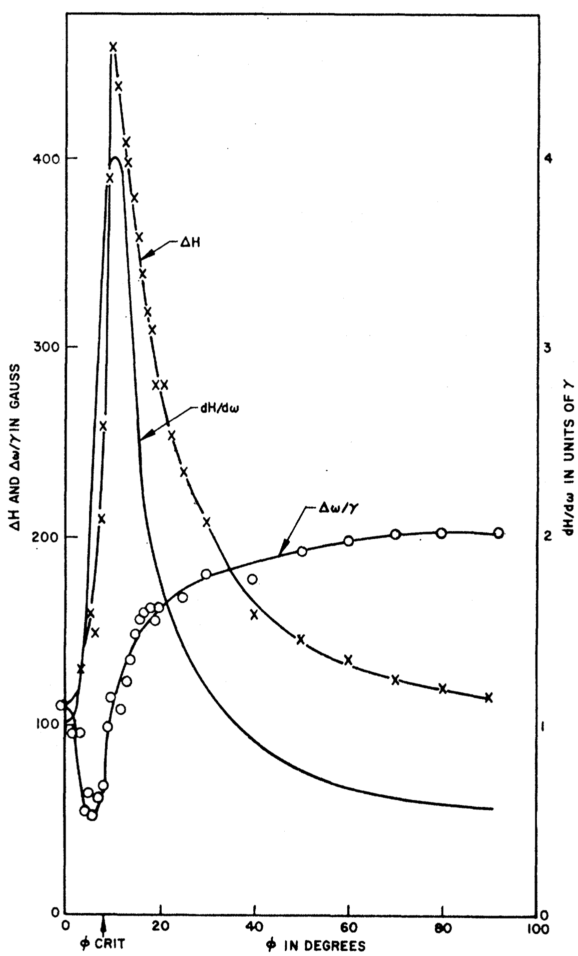


FIG. 6. Same as Fig. 5 for film PC22-2.

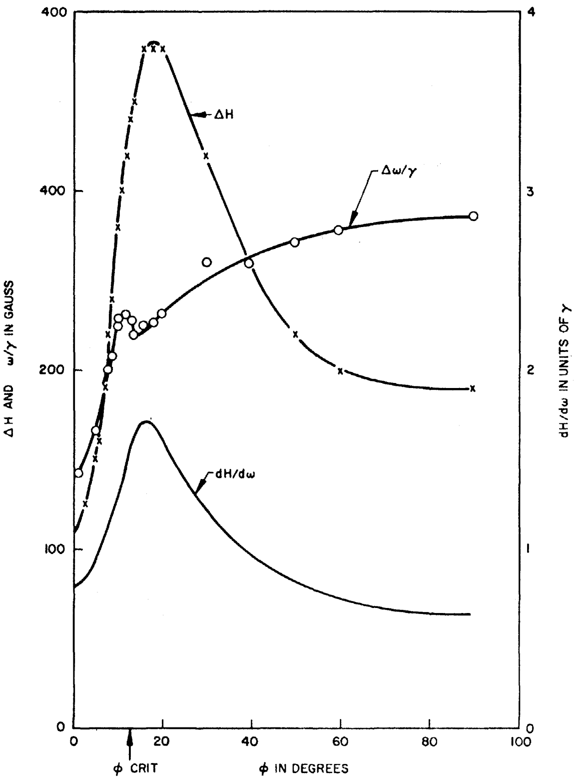


FIG. 7. Same as Fig. 5 for PC22-2 from data obtained at 14.6 kMc/sec.

theory. The theory assumes that the scattering occurs equally throughout the sample while in the thicker films the scattering may not take place as effectively in the central regions of the film where a minimum in the inhomogeneity is assumed.

## IV. ANGLE DEPENDENCE OF THE LINEWIDTH

While this calculation is not valid when the external field does not lie along the symmetry axis (normal to the film surface), a few relevant comments and observations can be made.

The critical angle in thin-film spin-wave resonance is that angle at which the resonant conditions are the same throughout the film and all the spins will precess at a common frequency in the same magnetic field.[1,3] Under these conditions the planar inhomogeneities will not be effective in broadening the excited mode and a decrease in the linewidth is expected.

The experimental results for the dependence of the linewidth on the angle are shown in Figs. 5–7. The damping parameters are related to the frequency linewidth $\Delta\omega$. From the dispersion relation in Eq. (1), it can be shown that the magnetic field linewidth $\Delta H$ measured in the rotation experiment is related to the frequency linewidth $\Delta\omega$ by an angle-dependent term $dH/d\omega$.[10]

$$\Delta H = (dH/d\omega)\Delta\omega. \tag{11}$$

The angle dependence of $dH/d\omega$, the experimental linewidths $\Delta H$ and the resulting value of $\Delta\omega$ for two films at 9.6 kMc/sec are shown in Figs. 5 and 6. The sharp decrease in $\Delta\omega$ at the critical angle ($\varphi=8°$) is expected if the inhomogeneous broadening is not effective at this angle. It should be noted that the minimum in $\Delta\omega$ occurs at the critical angle despite the fact that $\Delta H$ and $dH/d\omega$ have their maxima near $\varphi=11°$.

At 14.6 kMc/sec similar results are shown in Fig. 7; however, the dip is not as sharp nor as pronounced as observed in the 9.6-kMc/sec data.

## V. SUMMARY

It has been shown that the inhomogeneous broadening of the resonance mode in thin films is narrowed by the dipole forces to give linewidths on the order $H_p^2/4\pi M$, where $H_p$ is the root mean-square value of the internal field inhomogeneity. The spin-wave modes excited along the $\theta_\kappa=0$ spin-wave manifold boundary are scattered to longer wavelength $\theta_\kappa\neq 0$ spinwaves by a two magnon scattering process due to the magnetic inhomogeneities. The calculation has results similar to the dipole narrowed inhomogeneous broadening in polycrystalline ferrites[5,6] where the uniform mode is scattered by a similar process.

The shape-dependent term $I$, which enters the ferrite problem through the transverse demagnetization factor $N_t$, has its analog in thin films in the spin-wave mode term $J_\kappa$, which changes with the dispersion term $H_{n,\kappa}$. In thin films this term can be quantatively compared with experiments by comparing the results at the various spin-wave modes excited, a comparison that is not readily obtained in ferrite ellipsoids.

At the critical angle, the inhomogeneities in thin films become self-compensating and all the spins precess together in the uniform mode. This is indicated in the damping parameters by a minimum in the frequency linewidth at this critical angle.


## ACKNOWLEDGMENTS

The author acknowledges many helpful discussions with M. R. Sparks, C. F. Kooi, M. R. Shanabarger, G. F. Herrmann, J. I. Kaplan, and H. Ehrenreich. The aid of M. R. Shanabarger in the experimental measurements is also appreciated.


[10] T. D. Rossing, J. Appl. Phys. 34, 1133 (1963).